Data Processing Procedure for Fatigue Life Prediction of Spot Welded Joints Using a Structural Stress Method

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- Fatigue testing
  - Various joints: Spot weld, SPR, FSLW, GMAW, Adhesive joints, etc.
  - Strain-controlled fatigue, four-point bending fatigue, etc.
  - Round bar specimen, sheet specimen
  - Thermo-mechanical fatigue (TMF)

- FEA based fatigue life analysis
  - Stress-life approach (S-N)
  - Strain-life approach (E-N)
  - Structural stress method
Outline

• Challenge/project scope
• Rupp and Co-workers’ Structural Stress Method in nCode Designlife
• Nonlinear Generalized Reduced Gradient (GRG) Algorithm
• Data Processing Procedure
• Discussion and Conclusion
Challenge/project scope

• Nominal stress is mesh-dependent in FEA for spot welded joints. Instead, structural stress is widely used.
• Some empirical factors are added in order to consider the effect of different sheet thicknesses, nugget diameters, stress types etc.
• It is neccessary to find optimum empirical factors when the database changes.

Rupp and Co-workers’ Structural Stress Method in nCode Designlife

- The spot welded joint is represented by a circular plate model.
- The structural stress is calculated from forces and moments acting on a rigid kernel.
- FEA is used to obtain the forces and moments.

\[ \sigma_{r,\text{max}} = \frac{F_{x,y}}{\pi d s} = 1.744 \frac{F_z}{s^2} = 1.872 \frac{M_{x,y}}{ds^2} \]

Rupp and Co-workers’ Structural Stress Method in nCode Designlife

\[ \sigma_{stru} = \sigma_{\text{max}} (F_X) \cos \theta - \sigma_{\text{max}} (F_Y) \sin \theta + \sigma (F_Z) + \sigma_{\text{max}} (M_X) \sin \theta - \sigma_{\text{max}} (M_Y) \cos \theta \]

where

**Shear force**

\[ \sigma_{\text{max}} (F_X) = \frac{F_X}{\pi d t} \times SFFXY \times d^{DEFXY} \times t^{TEFXY} \]

\[ \sigma_{\text{max}} (F_Y) = \frac{F_Y}{\pi d t} \times SFFXY \times d^{DEFXY} \times t^{TEFXY} \]

**Axial force**

\[ \sigma (F_Z) = \frac{1.744 F_Z}{t^2} \times SFFZ \times d^{DEFZ} \times t^{TEFZ} \text{ if } F_Z > 0 \text{ or } 0 \text{ if } F_Z \leq 0 \]

**Bending moment**

\[ \sigma_{\text{max}} (M_X) = \frac{1.872 M_X}{d t^2} \times SFMXY \times d^{DEMXY} \times t^{TEMXY} \]

\[ \sigma_{\text{max}} (M_Y) = \frac{1.872 M_Y}{d t^2} \times SFMXY \times d^{DEMXY} \times t^{TEMXY} \]

9 empirical factors exist:

- Scale factor - \( SFFXY, SFFZ \) and \( SFMXY \)
- Diameter Exponent - \( DEFXY, DEFZ \) and \( DEMXY \)
- Thickness Exponent - \( TEFXY, TEFZ \) and \( TEMXY \)

Rupp and Co-workers’ Structural Stress Method in nCode Designlife

• Initial empirical factors suggested for steel in nCode Designlife

<table>
<thead>
<tr>
<th>Component</th>
<th>Factor</th>
<th>Diameter Exponent</th>
<th>Thickness Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>FX, FY</td>
<td>SFFXY = 1.0</td>
<td>DEFXY = 0.0</td>
<td>TEFXY = 0.0</td>
</tr>
<tr>
<td>MX, MY</td>
<td>SFMXY = 0.6</td>
<td>DEMXY = 0.0</td>
<td>TEMXY = 0.5</td>
</tr>
<tr>
<td>FZ</td>
<td>SFFZ = 0.6</td>
<td>DEFZ = 0.0</td>
<td>TEFZ = 0.5</td>
</tr>
</tbody>
</table>

• An optimization method is needed to find the optimum empirical factors when the database changes

Nonlinear Generalized Reduced Gradient (GRG) Algorithm

- Optimization:
  - Find the input values that can maximize or minimize a certain objective function under certain constraints.
  - Find $X_i$ to minimize $f(X_i)$

- Our problem:
  - Find the 9 empirical factors that can minimize the error (or scatter) in fatigue data.
  - Find $X_i$ to minimize $\text{Error}(X_i)$

Note: idea comes from Jaguar’s application for SPR in aluminum panels

Nonlinear Generalized Reduced Gradient (GRG) Algorithm

Minimize $f(X)$

Subject to

$h_j(X) = 0, j = 1, 2, ..., m$

$x_i^{(l)} \leq x_i \leq x_i^{(u)}, i = 1, 2, ..., n$

- Number of independent variables: $n-m$
- Number of dependent variables: $m$

The original variables $[X]$ can be partitioned into an independent set $[Y]$ and a dependent set $[Z]$:

$[X] = \begin{bmatrix} Y \\ Z \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{n-m} \\ z_1 \\ z_2 \\ \vdots \\ z_m \end{bmatrix}$
Nonlinear Generalized Reduced Gradient (GRG) Algorithm

• Consider the first variations of the objective function and constraints:

\[ df(X) = \nabla_Y^T f dY + \nabla_Z^T f dZ \]

\[ dh = [C]dY + [D]dZ \]

• By assuming \( dh = 0 \), it can be found

\[ dZ = -[D]^{-1}[C]dY \]

• Substituting \( dZ \) into \( df(X) \):

\[ df(X) = (\nabla_Y^T f - \nabla_Z^T [D]^{-1}[C])dY = G_R^TdY \]

• \( G_R \) is called the generalized reduced gradient, and gives the search direction.

• Up till now, the constrained optimization problem is transformed to a nonlinear programming problem without constraints. With a suitable step length, an optimum solution can be found after iterations.
Data Processing Procedure

- Spot welded joints fatigue data from Auto/Steel Partnership (ASP)

![Tensile shear diagram]

<table>
<thead>
<tr>
<th>Steel grade</th>
<th>Nugget diameter (mm)</th>
<th>Sheet thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSLA</td>
<td>7.4</td>
<td>1.78</td>
</tr>
<tr>
<td>DQSK</td>
<td>7.6</td>
<td>1.62</td>
</tr>
<tr>
<td>IF</td>
<td>7.6</td>
<td>1.63</td>
</tr>
<tr>
<td>RA830</td>
<td>7.26</td>
<td>1.38</td>
</tr>
<tr>
<td>DP600</td>
<td>7</td>
<td>1.50</td>
</tr>
<tr>
<td>DP800</td>
<td>7</td>
<td>1.60</td>
</tr>
<tr>
<td>MS1300</td>
<td>7.2</td>
<td>1.62</td>
</tr>
<tr>
<td>TRIP</td>
<td>7.23</td>
<td>1.60</td>
</tr>
</tbody>
</table>

![Coach peel diagram]

1. Create FEA models for specimens with different geometries.

- Mesh size: ~3mm
- 2D shell elements are used to represent both sheets and the spot weld nugget is modeled as a bar element with a diameter same as the weld nugget.
Data Processing Procedure

2. Extract the nodal forces and moments from the FEA results as shown in Table 2.

Table 2. Nodal forces and moments for unit load of the tensile shear FEA models.

<table>
<thead>
<tr>
<th>LS Thickness Combination</th>
<th>load (N)</th>
<th>D</th>
<th>S1</th>
<th>S2</th>
<th>Fx1</th>
<th>Fy1</th>
<th>Fz1</th>
<th>Mx1</th>
<th>My1</th>
<th>Mz1</th>
</tr>
</thead>
<tbody>
<tr>
<td>HLSA 1.78mm</td>
<td>1</td>
<td>7.4</td>
<td>1.78</td>
<td>1.78</td>
<td>1</td>
<td>0</td>
<td>0.023734</td>
<td>0</td>
<td>0.89541</td>
<td>0</td>
</tr>
<tr>
<td>DQSK 1.62mm</td>
<td>1</td>
<td>7.6</td>
<td>1.62</td>
<td>1.62</td>
<td>1</td>
<td>0</td>
<td>0.021607</td>
<td>0</td>
<td>0.826416</td>
<td>0</td>
</tr>
<tr>
<td>IF 1.63mm</td>
<td>1</td>
<td>7.6</td>
<td>1.63</td>
<td>1.63</td>
<td>1</td>
<td>0</td>
<td>0.021607</td>
<td>0</td>
<td>0.826416</td>
<td>0</td>
</tr>
<tr>
<td>RA830 1.38mm</td>
<td>1</td>
<td>7.26</td>
<td>1.38</td>
<td>1.38</td>
<td>1</td>
<td>0</td>
<td>0.018411</td>
<td>0</td>
<td>0.706131</td>
<td>0</td>
</tr>
<tr>
<td>DP600 1.50mm</td>
<td>1</td>
<td>7</td>
<td>1.5</td>
<td>1.5</td>
<td>1</td>
<td>0</td>
<td>0.020007</td>
<td>0</td>
<td>0.755438</td>
<td>0</td>
</tr>
<tr>
<td>DP800 1.60mm</td>
<td>1</td>
<td>7</td>
<td>1.6</td>
<td>1.6</td>
<td>1</td>
<td>0</td>
<td>0.02134</td>
<td>0</td>
<td>0.805428</td>
<td>0</td>
</tr>
<tr>
<td>MS1300 1.62mm</td>
<td>1</td>
<td>7.2</td>
<td>1.62</td>
<td>1.62</td>
<td>1</td>
<td>0</td>
<td>0.021607</td>
<td>0</td>
<td>0.826416</td>
<td>0</td>
</tr>
<tr>
<td>TRIP 1.60mm</td>
<td>1</td>
<td>7.23</td>
<td>1.6</td>
<td>1.6</td>
<td>1</td>
<td>0</td>
<td>0.021339</td>
<td>0</td>
<td>0.805428</td>
<td>0</td>
</tr>
</tbody>
</table>

3. Substitute the nodal forces and moments into the structural stress equation to calculate the maximum structural stress ($\sigma_{stru}$) based on the initial empirical factors.

$$\sigma_{stru} = \sigma_{max}(F_X)\cos \theta - \sigma_{max}(F_Y)\sin \theta + \sigma(F_Z) + \sigma_{max}(M_X)\sin \theta - \sigma_{max}(M_Y)\cos \theta$$
Data Processing Procedure

4. Plot the structural stress range versus test fatigue life (S-N curve), then use this S-N curve to calculate the predicted fatigue life for each structural stress range (black line).

5. The purpose is to reduce the scatter, so $R^2$ is the target to be maximized:

$$R^2 = 1 - \frac{SS_E}{SS_T}$$

$$SS_E = \sum_i (y_i - \hat{y}_i)^2, \quad SS_T = \sum_i (y_i - \bar{y})^2$$

$y_i$ - test life
$\hat{y}_i$ - predicted life
$\bar{y}$ - sample mean
$SS_T$ - constant
$SS_E$ - sum of error squares
6. Use the nonlinear GRG method to solve the optimization problem. Obtain the 9 optimized empirical factors.

Minimizing Error(\(X\))

subject to

\[ \sigma_j(X) \geq 0, \quad j = 1, 2, \ldots, m \]

\[ 0 \leq x_i \leq 1, \quad i = 1, 2, \ldots, 9 \]

• \(\sigma_j(X)\) - structural stress for each test
• \(x_i\) - 9 empirical factors to be determined

Table 3. Optimized empirical factors based on GRG algorithm

<table>
<thead>
<tr>
<th>Material</th>
<th>Scale factor</th>
<th>Diameter Exponent</th>
<th>Thickness Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SFFXY</td>
<td>SFFZ</td>
<td>DEFFZ</td>
</tr>
<tr>
<td>steel</td>
<td>0.9888</td>
<td>0.3059</td>
<td>0.5735</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.2369</td>
<td>0.6119</td>
</tr>
<tr>
<td></td>
<td>0.3871</td>
<td>0.6461</td>
<td>0.8206</td>
</tr>
</tbody>
</table>

7. Recalculate the structural stress based on the new empirical factors.
Discussion and Conclusion

- Data from ASP
- Scatter is reduced after using optimized empirical factors

S-N curve before optimization, $R^2=0.73$

S-N curve after optimization, $R^2=0.87$
Discussion and Conclusion

- Date tested in UM-Dearborn
- Scatter is reduced after using optimized empirical factors obtained from ASP’s data

S-N curve with initial empirical factors, $R^2=0.78$

S-N curve with empirical factors optimized from ASP’s data, $R^2=0.87$
Discussion and Conclusion

- Estimated specimen life correlates well with fatigue test life for different kinds of steel, specimen types, sheet thicknesses and nugget diameters

- Proposed optimization procedure for determining the empirical factors are validated

Predicted life versus experimental life
Discussion and Conclusion

- A data processing procedure is proposed to optimize the empirical factors in Rupp’s structural stress calculation.

- The optimized results show less degree of scatter than previous calculations with initial empirical factors.

- The fatigue life predicted using this procedure is well correlated with test results.
Thank you!